Propositions assignment

Marketing strategies have changed over the last decade from mass-marketing (all customers are targeted with one main message about a new product or promotion) to personnalized advertising, where specific products and messages are addressed to each customer, to offer the right product to the right customer who needs it.
This change has brought up a new problem to marketeers, which is to define how to distribute the available promotions to their customers in order to maximize the potential return of the marketing campaign.

A typical situation can be described as follows:

- the operator has a number of propositions (for example a product at a discounted price) that can be offered to their customers;
- for each proposition, the operator has a limited capacity that they can offer to their customers, the capacity may vary depending on the proposition;
- each proposition generates a return on investment, which may depend on the type of proposition;
- the operator computes for each customer and for each proposition the probability that the customer will accept the proposition if it is offered to him.
- some proposition cannot be combined with others (if a customer has received an offer for 2Gb of data at 10% discount, he can't receive after that an offer for 1Gb of data at 10% discount).

The operator needs then an assignment of x proposition per customer, in order to maximize the expected return (x may vary depending on the situation).

In this project, the students will be asked to

1. Formalize the problem as an optimization problem, keeping in mind all the possible constraints that are described above;
2. Implement and test an algorithm to solve exactly this problem on a small synthetic example;
3. Observe how the algorithm behaves when the size of the problem increases in size;
4. Suggest and implement one or more heuristics to solve this problem on larger instances (examples will be provided). In case the students propose several heuristics, it would be interesting to compare the performances of those both in terms of speed and quality of the solution.

Input Data

- The propensity matrix $\boldsymbol{P} \in \{0,1\}^{M \times N}$:
  p_{ij} is the propensity of customer i to accept proposition j
- The revenue matrix $\boldsymbol{V} \in \mathbb{R}^{M \times N}$:
  v_{ij} is the revenue associated to customer i accepting proposition j
- The set of mutually exclusives propositions indices $\mathcal{E}$:
  $\mathcal{E} = \{(e_1^1, e_2^1), \ldots, (e_1^L, e_2^L)\}$ where each pair denotes two mutually exclusives propositions
- Typical dimensions:
  $M = 10^6$ customers, $N = 100$ propositions, $L \leq 30$ forbidden combination of propositions

Optimization variables

- The assignment matrix $\boldsymbol{A}$:
  $a_{ij} = 1$ if proposition j is to be assigned to customer i, 0 otherwise